

$$PP = (p, g).$$

Schnorr Id is interactive protocol, but not recurrent as it realized to prove the mirckle words.

Schnorr Id Scenario: Alice wants to prove Bank that she knows her Private Key - $PrK_A = x$ which corresponds to her Public Key - $PuK_A = a = g^x \bmod p$ not revealing $PrK_A = x$.

A: Prover $P(x, a)$

ZKP of knowledge $PrK=x$:

1. Computes commitment

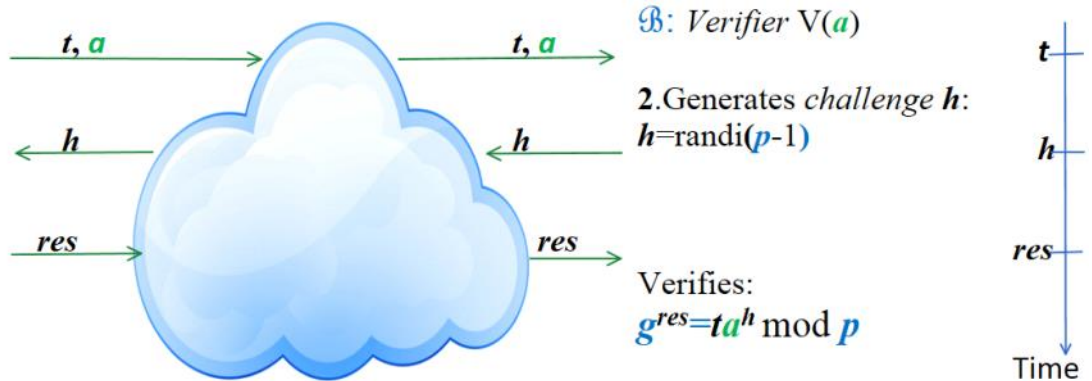
t for random number i :

$$i = \text{randi}(p-1)$$

$$t = g^i \bmod p$$

3. Computes response res :

$$res = i + xh \bmod (p-1)$$



Correctness:

$$g^{res} \bmod p = g^{i+xh \bmod (p-1)} \bmod p = g^i g^{xh} \bmod p = t(g^x)^h \bmod p = ta^h \bmod p.$$

```
>> p= int64(268435019);
>> g=2;
```

```
>> x=int64(randi(p-1))
x = 130007868
>> a=mod_exp(g,x,p)
a = 176162386
```

```
>> i=int64(randi(p-1))
i = 65140707
```

```
>> t=mod_exp(g,i,p)
t = 189627349
>> h=int64(randi(p-1))
h = 242654365
>> res=mod(i+x*h,p-1)
res = 20761443
```

```
>> res=mod(i+x*h,p-1)
res = 20761443
>> g_res=mod_exp(g,res,p)
g_res = 215652516
>> V1=g_res
V1 = 215652516
>> a_h=mod_exp(a,h,p)
a_h = 73404223
>> ta_h=mod(t*a_h,p)
ta_h = 215652516
>> V2=ta_h
V2 = 215652516
```

Alice chooses at random u , $1 < u < p-1$ and computes first component r of his signature:

$$r = g^u \bmod p. \tag{2.19}$$

Alice computes H-function value h and second component s of her signature:

$$h = H(M || r), \tag{2.20}$$

$$s = u + xh \bmod (p-1). \tag{2.21}$$

Alice's signature on h is $\sigma = (r, s)$. Then Alice sends M and σ to Bob.

After receiving M' and σ , **Bob** according to (2.20) computes h'

$$h' = H(M' || r),$$

and verifies if

$$\underset{V1}{g^s \bmod p} = \underset{V2}{ra^{h'} \bmod p}. \quad (2.22)$$

Symbolically this verification function we denote by

$$\mathbf{Ver}(a, \sigma, h') = V \in \{\mathbf{True}, \mathbf{False}\} \equiv \{\mathbf{1}, \mathbf{0}\}. \quad (2.23)$$

This function yields **True** if (2.22) is valid if: $h = h'$ and $\mathbf{PuK}_A = a = F(\mathbf{PrK}_A) = g^x \bmod p$.
and: $M = M'$

```
>> u=int64(randi(p-1))
u = 158865637
>> r=mod_exp(g,u,p)
r = 44519947
>> cc=concat(m,r)
cc = Hello Bob44519947
>> h=hd28(cc)
h = 27628438
>> s=mod((u+x*h),p-1)
s = 267483811

>> g_s=mod_exp(g,s,p)
g_s = 239508680
>> V1=g_s
V1 = 239508680
>> a_h=mod_exp(a,h,p)
a_h = 128538175
>> ra_h=mod(r*a_h,p)
ra_h = 239508680
>> v2=ra_h
v2 = 239508680
```

```
>> m='Hello Bob'
m = Hello Bob
>> u=int64(randi(p-1))
u = 228451192
>> r=mod_exp(g,u,p)
r = 33418907
>> cc=concat(m,r)
cc = Hello Bob33418907 % cc is a string type variable
>> cc=concat(m,'33418907')
cc = Hello Bob33418907
>> cc=concat(m,'r')
cc = Hello Bobr

>> h=hd28(cc)
h = 104824510
>> s=mod((u+x*h),p-1)
s = 147250342

>> g_s=mod_exp(g,s,p)
g_s = 185672370
V1=g_s;
>> a_h=mod_exp(a,h,p)
a_h = 263774143
>> V2=mod(r*a_h,p)
V2 = 185672370
```

RSA Blind signature

```
>> p=genprime(14)
p = 10069
>> dec2bin(p)
ans = 10011101010101
>> q=genprime(14)
q = 15541
>> dec2bin(q)
ans = 11110010110101

>> fy=(p-1)*(q-1)
fy = 156456720
>> d=mulinv(e,fy)
d = 105260993
>> mod(e*d,fy)
ans = 1
>> m=100
m = 100
```

```

q = 15541
>> dec2bin(q)
ans = 11110010110101
>> n=p*q
n = 156482329
>> dec2bin(q)
ans = 11110010110101
>> dec2bin(n)
ans = 1001010100111011101100011001
>> e=2^16+1
e = 65537

```

```

ans = 1
>> m=100
m = 100
>> t=int64(randi(n))
t = 61584152
>> gcd(t,n)
ans = 1
>> t_e=mod_exp(t,e,n)
t_e = 150571433
>> mm=mod(m*t_e,n)
mm = 34839716

```

$$m' = m \cdot t^e \pmod n \quad mm \equiv m' \xrightarrow{\sigma'}$$

$$\text{Ver}(Puk=(n,e), \sigma', m') = m' \xleftarrow{\sigma'}$$

B:

$$\begin{aligned} \text{Sign}(Pk=d, m') &= \sigma' \\ \sigma' &= (m')^d \pmod n = \\ &= (m \cdot t^e)^d \pmod n = \\ &= m^d \cdot t^{ed} \pmod n \xrightarrow{\text{mod } \phi} 1 \\ &= m^d \cdot t \pmod n \end{aligned}$$

```

>> SigB=mod_exp(mm,d,n)
SigB = 76295822
>>
>> mmm=mod_exp(SigB,e,n)
mmm = 34839716

```

$$\sigma' = m^d \cdot t \pmod n$$

A: unmaskes signed m'

$$\begin{aligned} (\sigma')^e \pmod n &= ((m')^d)^e \pmod n = (m')^{ed \pmod \phi} \pmod n = 1 \\ &= m' \pmod n = m' \quad \text{if } m' < n \Rightarrow \text{Signature is valid.} = \text{True} \end{aligned}$$

```

>> mmm=mod_exp(SigB,e,n)
mmm = 34839716

```

A extracts (unmasks) $m^d \pmod n = \sigma$ from σ' :
 $\sigma' \cdot t^{-1} \pmod n \rightarrow$ if $\text{gcd}(t, n) = 1 \Rightarrow t^{-1} \pmod n$ exists.
 $\sigma' \cdot t^{-1} \pmod n = \underbrace{m^d \cdot t}_{\sigma'} \cdot \underbrace{t^{-1}} \pmod n = \underbrace{m^d \pmod n}_{\sigma}$

```

>> SigBm=mod(SigB*t_m1,n)
SigBm = 143112175
>> m100=mod_exp(SigBm,e,n)
m100 = 100

```